Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV(OLD) EXAMINATION – WINTER 2022

Subject Code:140001 **Date:13-12-2022**

Subject Name:Mathematics-IV

Time:10:30 AM TO 01:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 **(a)** If $x_n = \cos\frac{\pi}{3^n} + i\sin\frac{\pi}{3^n}$, then show that **07**
$(a)\ i(x_1.x_2.x_3 \ldots x_\infty) = -1$ and $(b)\ i(x_0.x_1.x_2 \ldots x_\infty) = 1$

(b) Find a real root of the equation $x^3 + 4x^2 - 1 = 0$ by bisection method correct upto two decimal places. **07**

Q.2 **(a)** Find and plot all roots of $\sqrt[3]{8i}$ **07**

(b) State necessary and sufficient conditions for the function to be analytic. Also Show that $f(z) = |z|$ is not an analytic function. **07**

OR

(b) Define Harmonic function. Show that the function $u(x,y) = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic.Find the conjugate harmonic function v. **07**

Q.3 **(a)** Find the bilinear transformation that maps the points $z_1 = -2, z_2 = 0, z_3 = 2$ onto the points $w_1 = \infty, w_2 = \frac{1}{2}, w_3 = \frac{3}{4}$ respectively. **07**

(b) Evaluate $\int_C (x - y + ix^2)dz$, where C is along the line joining from $z = 0$ to $z = 1$, $z = 1$ to $z = 1 + i$ and $z = 1 + i$ to $z = 0$. **07**

OR

Q.3 **(a)** State Cauchy's integral formula. Also Evaluate $\oint_C \frac{e^z}{z(z-1)}dz$, where C is a circle $|z| = 2$. **07**

(b) Find the Laurent's series expansions of $f(z) = \frac{1}{(z+1)(z+3)}$ valid for $(a)\ |z| < 1\ \ (b)\ 1 < |z| < 3\ \ (c)\ |z| > 3$ **07**

Q.4 **(a)** Using Newton-Raphson method find a root of the equation $x^3 - 3x - 5 = 0$ correct upto four decimal places. **07**

(b) The population of a town in the census is as given in the data. Estimate the population in the year 1996. **07**

Year(x)	1961	1971	1981	1991	2001
Population(y) (in thousands)	46	66	81	93	101

OR

Q.4 **(a)** Compute the integral $I = \int_0^4 (x^3 - 2x^2 + 1)\,dx$ using Simpson's 1/3 rule taking $h = 1$. **07**

(b) Find an iterative formula to find square root of N (where N is a positive number) and hence find the square root of 8 correct upto two decimal places. **07**

Q.5 **(a)** Apply Gaussian elimination method to solve the following system of equations **07**

$x_1 + x_2 + 5x_3 = -1$, $2x_1 + 4x_2 = 12$, $5x_1 - x_2 + x_3 = 10$.

(b) Using Euler's method, find $y(0.04)$ for the following initial value problem. **07**

$y' = y$, $y(0) = 1$ .Take step size as $h = 0.01$.

OR

Q.5 **(a)** Solve the following system of equations by Gauss Seidel method. **07**

$10x_1 + x_2 + x_3 = 6$, $x_1 + 10x_2 + x_3 = 6$, $x_1 + x_2 + 10x_3 = 6$.

(b) Use the Runge-Kutta method to solve $\frac{dy}{dx} = -xy^2$ for $0 \leq x \leq 1$, subject to $y(0) = 2$. Use $h = 0.25$ and work for four decimal places. **07**

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